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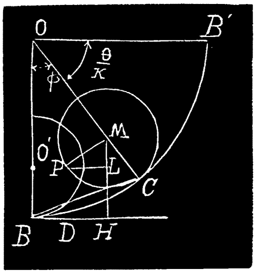
form of the solution here used is quite different from the usual method of trisection by the cardioid.

In this manner the curve corresponding to any value of n as above indicated can be found. By means of an instrument constructed on the principle which governs the relative rates of motion of the hands of a clock, any of these curves can be accurately drawn.

The hypocycloid curve may also be used for the purpose of general equi-section of an angle, as appears in the following:



Given OB the radius of a circle, and suppose $O'B=r=\dfrac{OB}{k.n}$ where $\dfrac{\theta}{360}=\dfrac{1}{n}$, and k is the number of equal parts into which it is required to divide the angle θ.

B and B' are two consecutive points common to the hypocycloid and the circumference of the circle whose center is O, the rolling circle having the radius $O'B=r$. It is evident that the angle $BOB'=\theta/k$. To find the equation of the curve, let B be the origin, BH the X-axis, BO the Y-axis, ϕ the angle BOC, P a point on the curve, $x=BD$ and $y=PD$. Then $y=MF+FH-ML=r+FH-ML$.

But $ML=r\cos(kn-1)\phi$, $FH=(BD+DH)\tan\frac{1}{2}\phi=(x+DH)\tan\frac{1}{2}\phi$, $DH=PL=r\sin(kn-1)\phi$.

Hence, $y=r+[x+r\sin(kn-1)\phi]\tan\frac{1}{2}\phi-r\cos(kn-1)\phi$, the equation of the hypocycloid, in which it may be noted that angle $PMC=k.n.\phi$.

While the equation is general, the construction of the curve can be made only when the ratio of the given angle to the perigon is known.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

290. Proposed by G. I. HOPKINS, M. A., Manchester, New Hampshire.

A and B are 45 miles apart and travel towards each other. A goes one mile the first day, three miles the second day, five miles the third day, and so on. B goes two miles the first day, four the second, six the third, and so on. In how many days will they meet? What interpretation is to be placed upon the negative value of n?

Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Let n=the required number of days for A and B to meet. When a,

d, and n are given in an arithmetical progression to find s we have

$$s=n[2a+d(n-1)]/2. \tag{1}$$

When for A we have $a=1$, and $d=2$, we have from (1), $s=n[2+2(n-1)]/2=n^2=$the distance traveled by A. When for B we have $a=2$ and $d=2$, we have from (1), $s=n[4+2(n-1)]/2=(n^2+n)=$the distance traveled by B.

$$\therefore 2n^2+n=45;\text{ whence } n=4.5 \text{ or } -5. \tag{2}$$

When $n=4.5$, $n^2=20.25=$distance traveled by A. We then have $(n^2+n)=24.75=$the distance traveled by B. Or $20.25+24.75=45$.

When $n=-5$, $n^2=25=$distance traveled by A. We then have $(n^2+n)=(-5)^2+(-5)=25-5=20=$the distance traveled by B. Or $25+20=45$.

The positive value, 4.5, and the negative value, -5, may be interpreted as follows: Draw a circle 90 miles in circumference, and let A stand at N, the north pole, and B at S, the south pole, 45 miles from each other, measured on either semi-circumference. In the 4½ days A will walk 20.25 miles from N along the eastern semi-circumference until he meets B who has walked 24.75 miles from S along the eastern semi-circumference. A's position is now southeast from N, 81°; and B's position is now northeast from S, 99°; or they are theoretically at the same point. In the -5 days, or 5 days ago, A and B were theoretically at the same point on the western circumference 100° from N or 80° from S, A walked towards N and traveled 25 miles, while B walked towards S and traveled 20 miles, then A is at N, and B at S.

NOTE. This problem was also solved by J. E. Sanders, who obtained as a result $4\frac{9}{19}$ days. Since at the end of the 4th day the two men are 9 miles apart, Mr. Sanders assumes that A, who would travel 10 miles the 5th day provided he kept up the law of travel of the four previous days, would, in the time, t, travel $10t$, and B on the same conditions would travel $9t$; and, therefore, both would travel $19t=9$ miles. Whence $t=\frac{9}{19}$. Hence the whole time is $4\frac{9}{19}$ days. Mr. Sanders interprets —5 days as the answer to the problem, "A and B are 45 miles apart and travel towards each other. A goes 1 mile the first day, 3 miles the second day, 5 miles the third, and so on. B goes 0 miles the first day, 2 miles the second day, 4 miles the third day, and so on." While Mr. Sanders' interpretation, as well as that of Mr. DeLand, is ingenious, and perhaps also correct, yet such interpretations seem to be *strained*. By the insertion of a few words, one would obtain the same set of results while the interpretations would fail to *interpret*.

Thus, suppose the men were restricted to walk on the straight line joining them. This would nullify Mr. DeLand's interpretation as to their traveling in a circle. In the case A and B are to travel in a straight line joining them, 4½ days as given by Mr. DeLand, is the most satisfactory, and he has furnished, in a letter to the editor, some very convincing arguments drawn from practical interpolation formulae to support his view. However, whatever interpretation lawyers and judges might put on similar problems involving similar principles when applied to government bonds, etc., yet the universal testimony of the best mathematicians would be to the effect that the problem is *indeterminate*, for the statement of the problem is silent as to the rates of travel of the two men on the respective days. Mr. Sanders assumes that, on the fifth day, they travel uniformly, while Mr. DeLand assumes that A travels 4.75 miles, and B 4.25 miles in the forenoon, and were they to keep up the law of travel as on previous days A would travel 5.25 miles and B 4.75 miles in the afternoon. Any other conceivable rate of travel each day might be assigned. The problem only requires them to travel a certain number of miles each successive day, but leaves the rates of travel to the ingenuity of the interpreter.

As to negative results, there is some diversity of opinion among good algebraists. Thus, for example, C. Smith, *Treatise on Algebra*, p. 259, says, "It should be remarked that a negative value of n cannot mean a number of terms reckoned *backwards*;" while Hall and Knight, *Higher Algebra*, page 33, in their solution of the problem, "How many terms of the series —9, —6, —3, ..., must be taken that the sum may be 66," say, "If we begin at the last of these terms and *count backwards* four terms, the sum is also 66."

Fine, *College Algebra*, page 355, gives the following example: "Given $d=\frac{1}{2}$, $l=\frac{3}{2}$, $s=-\frac{15}{2}$; find a and n." In reference to the two values of n, viz., 10 and —3, Professor Fine says, "The value of $n=-3$ is inadmissible." So say Fisher and Schwatt, *Higher Algebra*, page 382. But they say such results may be assumed to mean that the terms be counted backwards.

The weight of authority seems to be that in such problems negative and fractional values are uninterpretable. Ed. F.

GEOMETRY.

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324. Proposed by FRANK LOXLEY GRIFFIN, S. M., Ph. D., Instructor in Mathematics, Williams College, Williamstown, Mass.

Find all plane curves such that the normal lengths intercepted by the co-ordinate axes are in a constant ratio for all points.

Solution by PROFESSOR C. E. WHITE, Nashville, Tenn., and the PROPOSER.

Let $f(x, y)=0$ be the equation of the required curves. Then the normal at the point (x, y) has the equations:

$$\frac{\xi-x}{\frac{\partial f}{\partial x}}=\frac{\eta-y}{\frac{\partial f}{\partial y}}=\frac{-\rho}{R},$$

where (ξ, η) is any point of the normal, ρ the distance from (x, y) to (ξ, η), and $R=\sqrt{\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}$. The normal length intercepted by the axes, $x=0$, and $y=0$, are the values of ρ for $\xi=0$, and for $\eta=0$, respectively. Thus,

$$\rho_x=x.\frac{R}{\frac{\partial f}{\partial x}} \text{ and } \rho_y=y.\frac{R}{\frac{\partial f}{\partial y}}.$$

By hypothesis, $k_1\rho_x=k_2\rho_y$, or

$$\frac{k_1x}{\frac{\partial f}{\partial x}}=\frac{k_2y}{\frac{\partial f}{\partial y}}. \tag{1}$$

But, since $\frac{\partial f}{\partial x}.dx+\frac{\partial f}{\partial y}.dy=0$,

$$k_1xdx+k_2ydy=0, \tag{2}$$

whence, c being an arbitrary constant,

$$k_1x^2+k_2y^2=c. \tag{3}$$